



NPTEL ONLINE CERTIFICATION COURSES

FINITE ELEMENT METHOD

Biswanath Banerjee & Amit Shaw

Department of Civil Engineering, IIT Kharagpur

Module 05: FEM for Timoshenko Beam

Lecture 22 : Shear Deformable Beam Theory

Concepts Covered

- Shear Deformable Beam - Kinematics
- Stresses – Shear correction factor
- Strain Energy and Governing equation
- Classical Rayleigh-Ritz Method





**THANK
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Lecture 23 : Weak Form and Discretization

Concepts Covered

- **Weak Form**
- **Shear Locking – Thin beam**
- **Consistent interpolation and Stiffness matrix**





**THANK
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Lecture 24 : Reduced Integration Based Stiffness Matrix

Concepts Covered

- Reduced Integration – Same order of interpolation
- Stiffness Matrix





**THANK
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Lecture 25 : Problem

Concepts Covered

- A Simple Problem

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Lecture 26 : Problem

Concepts Covered

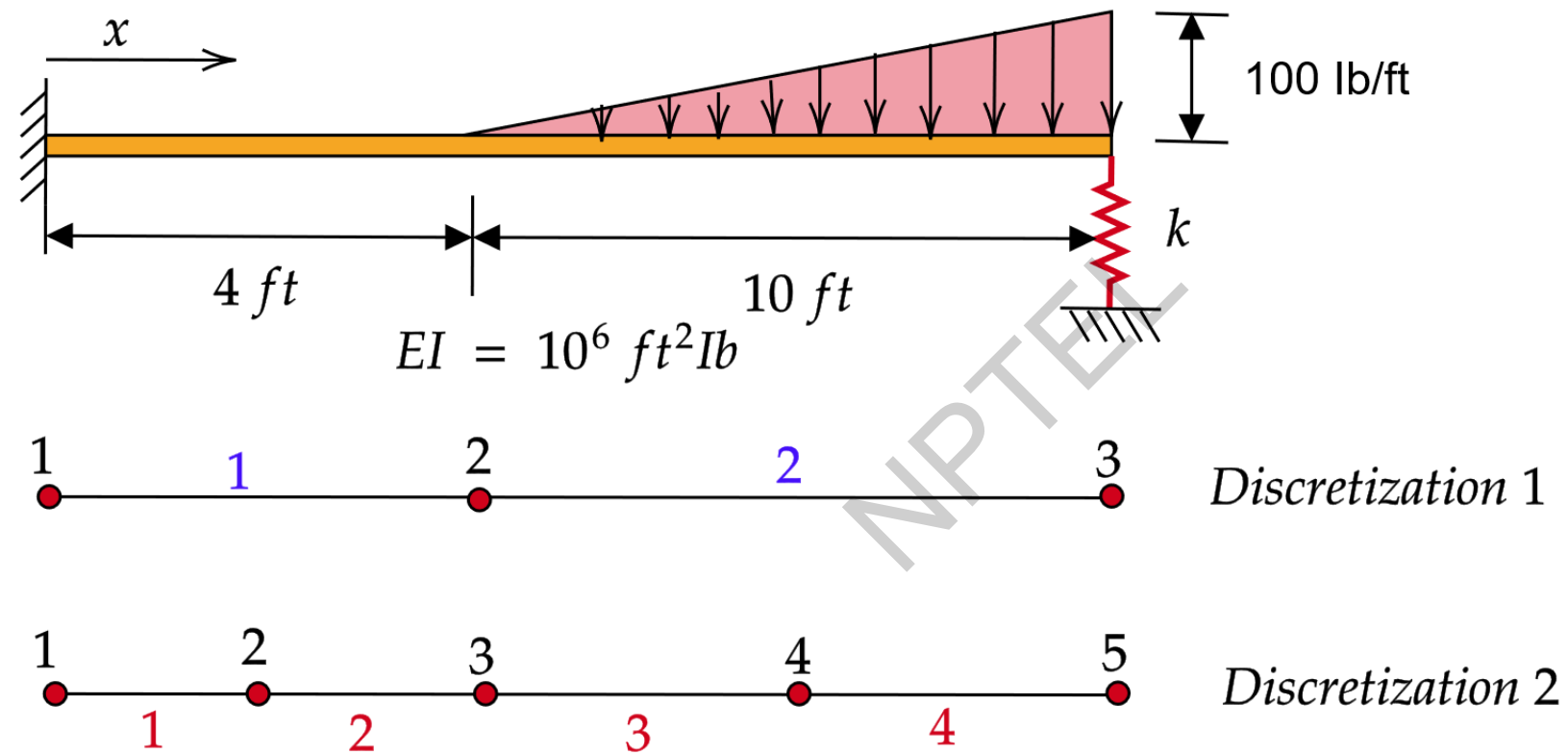
- Problem

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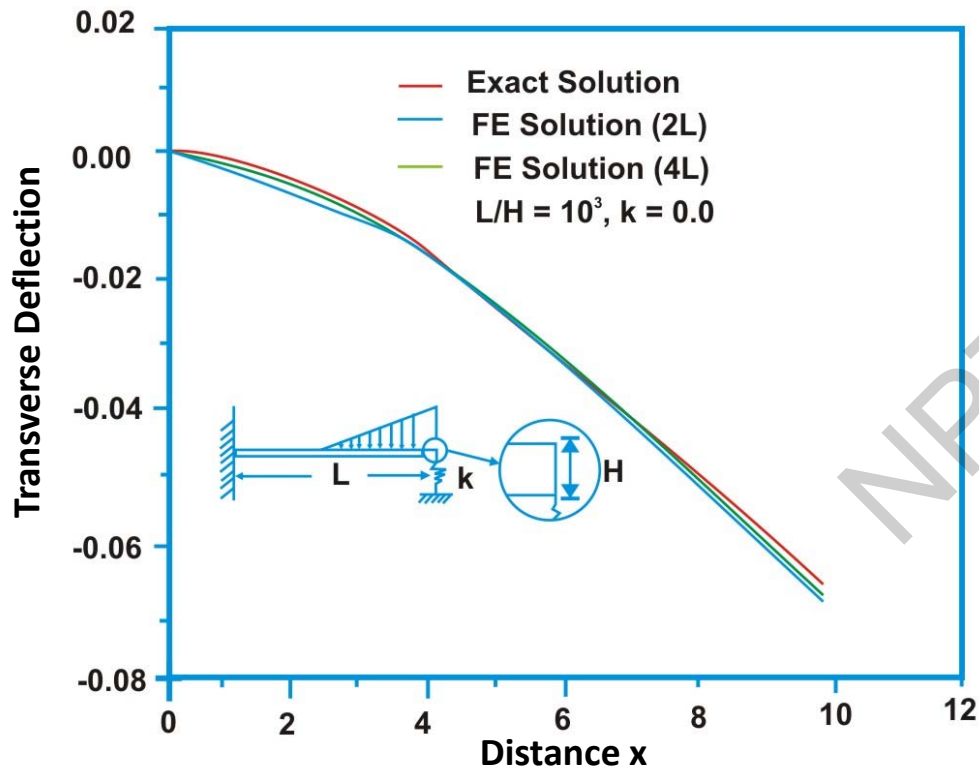


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Problem



Case 1: Cantilever Thin Beam ($k = 0$)

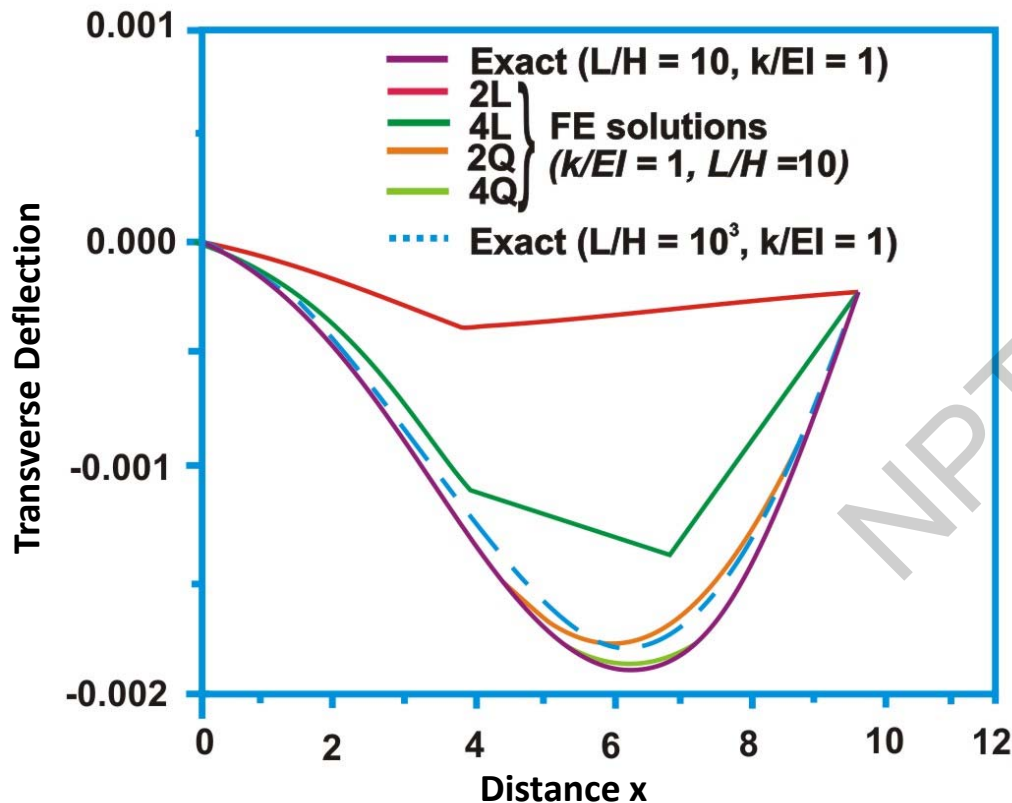


- Reduced integration Element (RIE)
- L/H is large i.e. thin beam

Two node linear Timoshenko beam element

$$w(\bar{x}) = \left(1 - \frac{\bar{x}}{L_e}\right) w_1 + \frac{\bar{x}}{L_e} w_2$$
$$\theta(\bar{x}) = \left(1 - \frac{\bar{x}}{L_e}\right) \theta_1 + \frac{\bar{x}}{L_e} \theta_2$$

Case 2a: Beam with $k = EI$



- Reduced integration Element (RIE)
- L/H is not large i.e. not a thin beam

Two node linear Timoshenko beam element

$$w(\bar{x}) = \left(1 - \frac{\bar{x}}{L_e}\right) w_1 + \frac{\bar{x}}{L_e} w_2$$

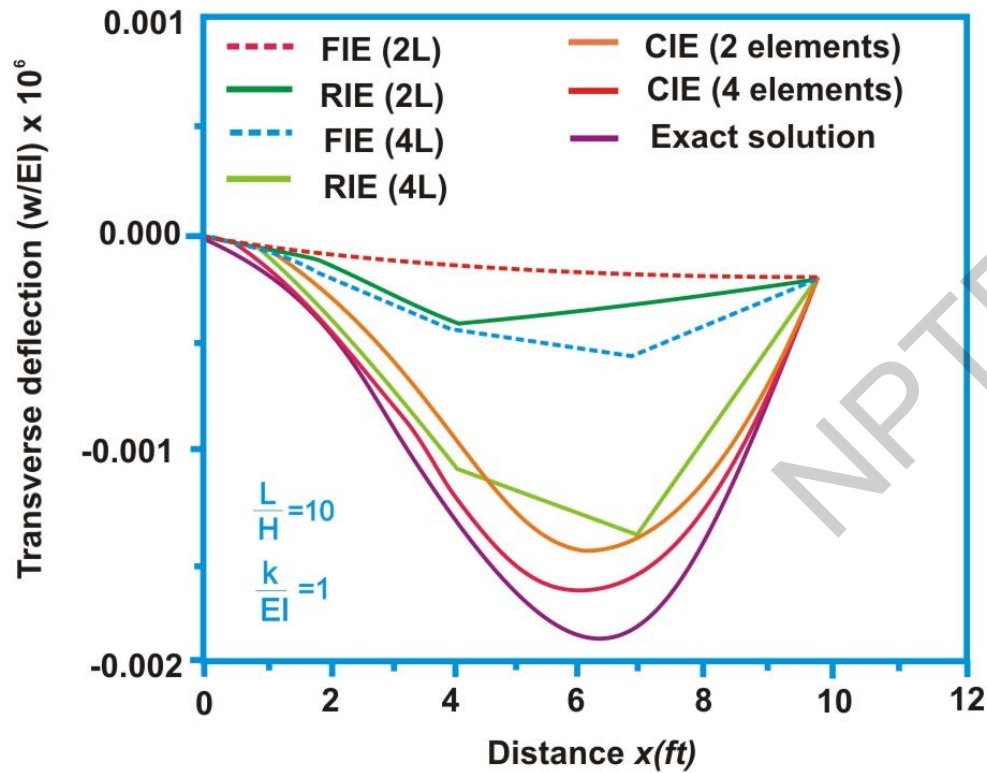
$$\theta(\bar{x}) = \left(1 - \frac{\bar{x}}{L_e}\right) \theta_1 + \frac{\bar{x}}{L_e} \theta_2$$

Three node quadratic Timoshenko beam element

$$w(\bar{x}) = \varphi_1 w_1 + \varphi_2 w_2 + \varphi_3 w_3$$

$$\theta(\bar{x}) = \varphi_1 \theta_1 + \varphi_2 \theta_2 + \varphi_3 \theta_3$$

Case 2b: Beam with $k = EI$



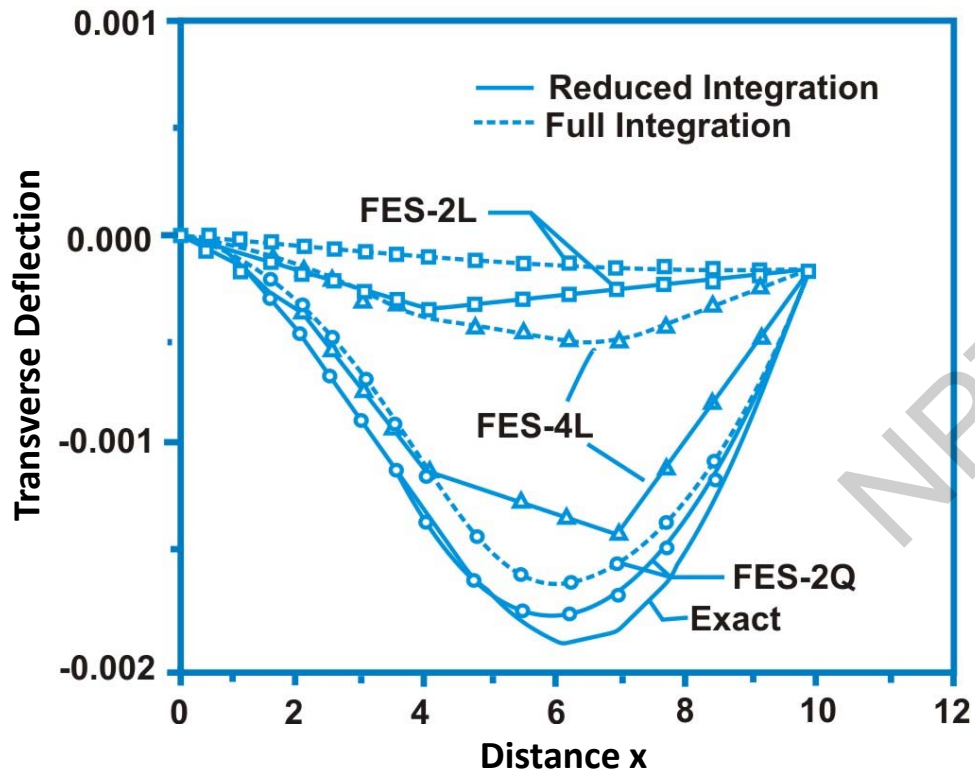
- L/H is not large i.e. not a thin beam

- Full integration Element (FIE)
- Reduced integration Element (RIE)
- Consistent Interpolation Element (CIE)

- CIE: Displacement is interpolated Quadratically

$$w(\bar{x}) = \varphi_1 w_1 + \varphi_2 w_2 + \varphi_3 w_3$$
$$\theta(\bar{x}) = \psi_1 \theta_1 + \psi_2 \theta_2$$

Case 3: Beam with $k = EI$



- L/H is not large i.e. not a thin beam

- Full integration Element (FIE)
- Reduced integration Element (RIE)



**THANK
YOU !**